EXAMEN BLOQUE II - GEOMETRÍA

TEST

1.-

$A(1,1,1) \quad B(2,2,2) \quad C(0,1,1)$

$\vec{AB} = (2,2,2) - (1,1,1) = (1,1,1)$

$\vec{AC} = C - A = (-1,0,0)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{vmatrix} = 0\vec{i} - \vec{j} + \vec{k}$$

$|\vec{AB} \times \vec{AC}| = \sqrt{0^2 + (-1)^2 + (1)^2} = \sqrt{2} \rightsquigarrow$ Área triángulo $= \frac{1}{2}\sqrt{2} \Rightarrow$ c

2.-

$r: x-1 = y+1 = z-2 \qquad s: x+2 = \frac{y+2}{2} = \frac{z-1}{4}$

$\vec{V_r} = (1,1,1) \qquad \vec{V_s} = (1,2,4) \qquad \vec{P_rP_s} = (-3,-1,-1)$

$P_r = (1,-1,2) \qquad P_s = (-2,-2,1)$

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ -3 & -1 & -1 \end{pmatrix} \rightsquigarrow |M| = \begin{array}{c} + \qquad - \\ \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ -3 & -1 & -1 \end{vmatrix} \\ \begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{matrix} \end{array} = -2 - 1 - 12 - (-6) - (-4) - (-1)$$

$= -4 \neq 0 \longrightarrow Rg\, M = 3$

- Si $Rg\,M = 3 \rightsquigarrow$ se cruzan $\Rightarrow$ a

3.-

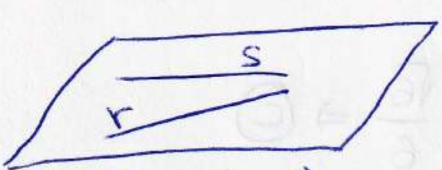
$r: x-1 = y+1 = z-2 \qquad s: x+2 = \frac{y+2}{2} = \frac{z-1}{4}$

$\vec{V_r} = (1,1,1) \qquad P(0,0,0)$

$\vec{V_s} = (1,2,4)$

$$\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = 2x - 3y + z = 0 \Rightarrow \text{a}$$

Forman un plano



$\vec{V_1} = \vec{V_r} \qquad \vec{V_2} = \vec{V_s}$

4.-

$\pi: x + y - 2z = -1$ $\qquad$ $r: x-1 = -y = z$

↳ El vector es -1

$$r: \begin{cases} x = 1+\lambda \\ y = -\lambda \\ z = \lambda \end{cases}$$

$$(1+\lambda) + (-\lambda) - 2(\lambda) = -1$$

$$1 + \cancel{\lambda} - \cancel{\lambda} - 2\lambda = -1$$

$$-2\lambda = -2$$

$\lambda = 1 \rightarrow$ se cortan $\Rightarrow$ b

5.-

$A(1,0,0)$ $\quad$ $B(1,1,1)$ $\quad$ $C(2,-1,1)$

$\vec{AB} = (0,1,1)$ $\quad$ $P(1,0,0)$ $\quad$ $\begin{vmatrix} x-1 & y & z \\ 0 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$

$\vec{AC} = (1,-1,1)$

$$\begin{vmatrix} x-1 & y & z \\ 0 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 2(x-1) + y - z = 0$$

$2x - 2 + y - z = 0 \rightarrow 2x + y - z = 2 \Rightarrow$ a

6.-

$\pi_1: x + 2y + z - 1 = 0$ $\qquad$ $\pi_2: x + 2y + z - 3 = 0$

$$d(\pi_1, \pi_2) = \frac{|D_1 - D_2|}{\sqrt{A^2+B^2+C^2}} = \frac{|-1-3|}{\sqrt{1^2+2^2+1^2}} = \frac{4}{\sqrt{6}}\,u = \frac{4\sqrt{6}}{6} = \frac{\sqrt{6}}{3}\,u \Rightarrow \text{c}$$

7.-

$P(1,0,1)$ $\quad$ $Q(2,1,-1)$ $\quad$ $R(1,2,1)$

$\vec{PQ} = (1,1,-2)$ $\qquad$ $\vec{PQ} \cdot \vec{PR} = |\vec{PQ}| \cdot |\vec{PR}| \cdot \cos\alpha$

$\vec{PR} = (0,2,0)$

$$\cos\alpha = \frac{0+2+0}{\sqrt{6}\cdot\sqrt{4}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} \Rightarrow \text{c}$$

8.-

$\vec{v}_1 = (2,-1,0)$ $\quad \vec{v}_2 = (1,2,1)$ $\quad \vec{v}_3 = (3,1,1)$

$$\begin{vmatrix} 2 & -1 & 0 \\ 1 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 4-3+0-0-2-(-1) = 4-3-2+1 = 0$$

Estan en el mismo plano

• Si están en el mismo plano son L.D $\Rightarrow$ b

9.-

$A(2,0,0)$ $\quad B(1,-2,0)$ $\quad C(0,1,2)$ $\quad D(x,y,z)$

$\vec{AB} = (-1,-2,0)$ $\qquad \vec{AB} = \vec{CD}$

$\vec{CD} = (x, y-1, z-2)$ $\qquad (-1,-2,0) = (x, y-1, z-2)$

$-1 = x$

$-2 = y-1$

$y = -1$

$0 = z-2$

$z = 2$

$D = (-1,-1,2) \rightarrow$ c

10.-

$\vec{v}_1 = (1,2,0)$ $\quad \vec{v}_2 = (-1,0,1)$ $\quad \vec{v}_3 = (-2,-1,3)$

a) $\vec{v}_1 \cdot \vec{v}_2 = 1\cdot(-1) + 2\cdot 0 + 0\cdot 1 = -1 \neq -4$

b) $[\vec{v}_1, \vec{v}_2, \vec{v}_3] = \begin{vmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ -2 & -1 & 3 \end{vmatrix} = 0-4+0-0-(-1)-(-6) = 3 \neq 6$

c) $\vec{v}_1 \times \vec{v}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ -2 & -1 & 3 \end{vmatrix} = 6\vec{i} - 3\vec{j} + 3\vec{k} = (6,-3,3) \Rightarrow$ c

11.-

$$r: \begin{cases} x = 2t + 1 \\ y = 3t - 1 \\ z = 1t - 7 \end{cases} \qquad \vec{v}_r = (2, 3, 1) \Rightarrow (b)$$

12.-

$$A(1,2,5) \qquad \pi: \overset{A}{2}x + \overset{B}{2}y \overset{C}{-} z - 5 = 0$$

$$d(A,\pi) = \frac{|A \cdot P_1 + B \cdot P_2 + C \cdot P_3 + D|}{\sqrt{A^2+B^2+C^2}} = \frac{|2 \cdot 1 + 2 \cdot 2 - 5 - 5|}{\sqrt{2^2+2^2+(-1)^2}} = \frac{|-4|}{\sqrt{9}} = \frac{4}{3} \Rightarrow (c)$$

13.-

$$r: \frac{x-2}{1} = \frac{y-3}{3} = \frac{z-1}{1} \qquad s: \frac{x-2}{1} = \frac{y-k}{1} = \frac{z-2}{2}$$

$$r: \begin{cases} x = 2+\lambda \\ y = 3+3\lambda \\ z = 1+\lambda \end{cases} \qquad s: \begin{cases} x = 2+\mu \\ y = k+\mu \\ z = 2+2\mu \end{cases}$$

si se cortan:

$$\begin{bmatrix} 2+\lambda = 2+\mu \\ 3+3\lambda = k+\mu \\ 1+\lambda = 2+2\mu \end{bmatrix} \begin{array}{l} \to \lambda = \mu \\ \to 3+3\lambda = k+\lambda \to k = 3+2\lambda \\ \to 1+\lambda = 2+2\lambda \end{array}$$

$$\boxed{-1 = \lambda} \qquad k = 3-2 \qquad \boxed{k=1}$$

- Si $k=1$ se cortan $\Rightarrow$ (b)

14.-

$$r: \begin{cases} 4x - y - z = 3 \\ 2x - z = 1 \end{cases} \qquad \text{si } \boxed{x = \lambda}$$

$$4\lambda - y - (2\lambda - 1) = 3$$

$$4\lambda - y - 2\lambda + 1 = 3$$

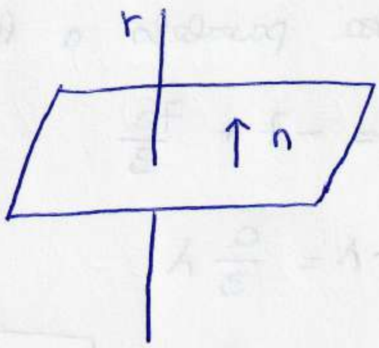
$$\boxed{y = 2\lambda - 2}$$

$$\to \boxed{z = 2\lambda - 1} \Rightarrow (b)$$

15.-

$r: x = y - 1 = z$

$P(1,1,1)$



$\vec{v_r} = \vec{n} = (A, B, C)$

$\vec{v_r} = (1, 1, 1)$

$\pi: Ax + By + Cz + D = 0 \longrightarrow x + y + z - 3 = 0 \Rightarrow$ a

$1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 + D = 0$

$D = -3$

PROBLEMAS

• OPCIÓN A:

1

$\pi: x - 3y + az = -6 \qquad r: \begin{cases} 2x - 3y = 1 \\ x + 3z = -7 \end{cases}$

• Pasamos la recta r a paramétrica: $x = \lambda$

$$\begin{cases} 2\lambda - 3y = 1 \longrightarrow y = \dfrac{2\lambda - 1}{3} = \dfrac{2}{3}\lambda - \dfrac{1}{3} \\ \lambda + 3z = -7 \longrightarrow z = \dfrac{-7 - \lambda}{3} = -\dfrac{7}{3} - \dfrac{\lambda}{3} \end{cases}$$

$$r: \begin{cases} x = \lambda \\ y = -1/3 + 2/3\lambda \\ z = -7/3 - 1/3\lambda \end{cases}$$

• Posición entre π y r: $\lambda - 3(-1/3 + 2/3\lambda) + a(-7/3 - 1/3\lambda) = -6$

$$\lambda + 1 - 2\lambda - \frac{7a}{3} - \frac{a}{3}\lambda = -6$$

$$-\lambda - \frac{a}{3}\lambda = -6 - 1 + \frac{7a}{3}$$

- Para que el plano sea paralelo a la recta: $0\lambda = k$

$$-\lambda - \frac{a}{3}\lambda = -7 + \frac{7a}{3}$$

$$-\lambda - \frac{a}{3}\lambda = 0 \rightsquigarrow -\lambda = \frac{a}{3}\lambda$$

$$-1 = \frac{a}{3} \rightsquigarrow \boxed{a = -3}$$

Comprobamos que para $a = -3$ da $0\lambda = k$

$$-\lambda - \frac{(-3)}{3}\lambda = -7 + \frac{7(-3)}{3}$$

$$-\lambda + \lambda = -7 - 7$$

$$0\lambda = -14 \quad ✓$$

2

$$r: \begin{cases} -x - 2y + 12 = 0 \\ 3y - z - 15 = 0 \end{cases} \qquad s: \frac{x-2}{5} = \frac{y+3}{2} = \frac{z}{3}$$

- Pasamos las rectas a paramétrica:

$$r: \; y = \lambda \longrightarrow \begin{cases} -x - 2\lambda + 12 = 0 \longrightarrow x = 12 - 2\lambda \\ 3\lambda - z - 15 = 0 \longrightarrow z = -15 + 3\lambda \end{cases}$$

$$r: \begin{cases} x = 12 - 2\lambda \\ y = \lambda \\ z = -15 + 3\lambda \end{cases} \qquad s: \begin{cases} x = 2 + 5\mu \\ y = -3 + 2\mu \\ z = 3\mu \end{cases}$$

a)

$$\left.\begin{matrix} \vec{v_r} = (-2, 1, 3) & P_r = (12, 0, -15) \\ \vec{v_s} = (5, 2, 3) & P_s = (2, -3, 0) \end{matrix}\right\} \overrightarrow{P_rP_s} = (-10, -3, 15)$$

$$M = \begin{pmatrix} -2 & 1 & 3 \\ 5 & 2 & 3 \\ -10 & -3 & 15 \end{pmatrix} \longrightarrow \text{Calculamos } Rg\,M$$

$$|M| = \begin{vmatrix} -2 & 1 & 3 \\ 5 & 2 & 3 \\ -10 & -3 & 15 \end{vmatrix} = -60-30-45+60+18-75 = -132 \neq 0 \rightarrow Rg\,M = 3$$

si es $Rg\,M = 3 \Rightarrow$ las rectas se cruzan

b)

$$d(r,s) = \frac{|[\vec{V_r}, \vec{V_s}, \vec{P_rP_s}]|}{|\vec{V_r} \times \vec{V_s}|} = \frac{|-132|}{\sqrt{531}} = \boxed{5,73\,u}$$

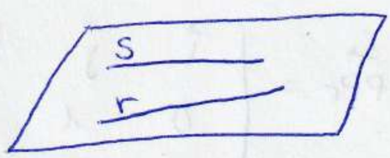
$$[\vec{V_r}, \vec{V_s}, \vec{P_rP_s}] = \begin{vmatrix} -2 & 1 & 3 \\ 5 & 2 & 3 \\ -10 & -3 & 15 \end{vmatrix} = -132$$

$$\vec{V_r} \times \vec{V_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 1 & 3 \\ 5 & 2 & 3 \end{vmatrix} = -3\vec{i} + 21\vec{j} - 9\vec{k} = (-3, 21, -9)$$

$$|\vec{V_r} \times \vec{V_s}| = \sqrt{(-3)^2 + 21^2 + (-9)^2} = \sqrt{531}$$

c)

Plano que contiene a r y s



$\vec{V_r} = \vec{V_1}$ $\quad P = P_s \quad$ $\vec{V_1} = (-2, 1, 3)$ $\quad P_s = (2, -3, 0)$

$\vec{V_s} = \vec{V_2}$ $\quad$ $\vec{V_2} = (5, 2, 3)$

$$\begin{vmatrix} x-2 & y+3 & z \\ -2 & 1 & 3 \\ 5 & 2 & 3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y+3 & z \\ -2 & 1 & 3 \\ 5 & 2 & 3 \end{vmatrix} = 0$$

$$(x-2)\cdot(-3) - (y+3)\cdot(-21) + z\cdot(-9) = 0$$

$$-3x + 6 + 21y - 63 - 9z = 0$$

$$\boxed{\pi: -3x + 21y - 9z = 57}$$

• OPCIÓN B:

1

$$r: (x, y, z) = (1,0,0) + t(0,1,1) \qquad P = (1,0,1)$$

a)

$$d(P,r) = \frac{|\vec{V_r} \times \vec{PP_r}|}{|\vec{V_r}|} = \frac{1}{\sqrt{2}} = \boxed{0{,}7\ u}$$

$$\vec{V_r} = (0,1,1) \longrightarrow |\vec{V_r}| = \sqrt{0^2+1^2+1^2} = \sqrt{2}$$

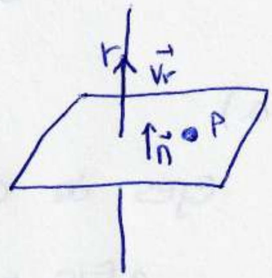
$$\left.\begin{array}{l} P_r = (1,0,0) \\ P = (1,0,1) \end{array}\right\} \vec{PP_r} = (0,0,1)$$

• $$\vec{V_r} \times \vec{PP_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i} = (1,0,0)$$

$$|\vec{V_r} \times \vec{PP_r}| = \sqrt{1^2+0^2+0^2} = 1$$

b)

P' simétrico de P respecto de r



• Plano perpendicular a r que pasa por P

$\vec{v_r} = \vec{n} = (0,1,1) \quad P = (1,0,1)$

$Ax + By + Cz + D = 0 \Rightarrow \boxed{y + z - 1 = 0}$

$\cancel{0 \cdot 1} + \cancel{1 \cdot 0} + 1 \cdot 1 + D = 0$

$D = -1$

• Intersección entre $r: \begin{cases} x = 1 \\ y = \lambda \\ z = \lambda \end{cases}$ y $\pi: y + z - 1 = 0$

$\lambda + \lambda - 1 = 0$

$2\lambda = 1$

$\lambda = 1/2 \longrightarrow \left.\begin{array}{l} x = 1 \\ y = 1/2 \\ z = 1/2 \end{array}\right\} P_0 = (1, 1/2, 1/2)$

• P_0 es el punto medio entre P y P':

$M = \frac{P + P'}{2} \longrightarrow (1, 1/2, 1/2) = \left(\frac{1+x}{2}, \frac{0+y}{2}, \frac{1+z}{2} \right)$

$\left.\begin{cases} 1 = \frac{1+x}{2} \longrightarrow x = 1 \\ 1/2 = \frac{y}{2} \longrightarrow y = 1 \\ 1/2 = \frac{1+z}{2} \longrightarrow z = 0 \end{cases}\right\} \boxed{P' = (1, 1, 0)}$

②

$\pi: x + 2y - 2z - 4 = 0$

a)

Ejes de coordenadas:

- Eje x: $\begin{cases} x = \lambda \\ y = 0 \\ z = 0 \end{cases}$ • Eje y: $\begin{cases} x = 0 \\ y = \lambda \\ z = 0 \end{cases}$ • Eje z: $\begin{cases} x = 0 \\ y = 0 \\ z = \lambda \end{cases}$

- $A \left\langle \begin{matrix} \pi \\ \text{eje } x \end{matrix} \right\} \quad \lambda + 2\cdot 0 - 2\cdot 0 - 4 = 0 \quad \lambda = 4 \Rightarrow \begin{matrix} x = 4 \\ y = 0 \\ z = 0 \end{matrix} \Rightarrow A = (4,0,0)$

- $B \left\langle \begin{matrix} \pi \\ \text{eje } y \end{matrix} \right\} \quad 0 + 2\cdot\lambda - 2\cdot 0 - 4 = 0 \quad \lambda = 2 \Rightarrow \begin{matrix} x = 0 \\ y = 2 \\ z = 0 \end{matrix} \Rightarrow B = (0,2,0)$

- $C \left\langle \begin{matrix} \pi \\ \text{eje } z \end{matrix} \right\} \quad 0 + 2\cdot 0 - 2\cdot\lambda - 4 = 0 \quad \lambda = -2 \Rightarrow \begin{matrix} x = 0 \\ y = 0 \\ z = -2 \end{matrix} \Rightarrow C = (0,0,-2)$

b)

$A = (4,0,0)$ $\quad \vec{AB} = (-4, 2, 0)$

$B = (0,2,0)$ $\quad \vec{AC} = (-4, 0, -2)$

$C = (0,0,-2)$ $\quad \vec{AP} = (-4, 0, 0)$

$P = (0,0,0)$

$$[\vec{AB}, \vec{AC}, \vec{AP}] = \begin{vmatrix} -4 & 2 & 0 \\ -4 & 0 & -2 \\ -4 & 0 & 0 \end{vmatrix} = 16 \rightsquigarrow V_\Delta = \frac{16}{6} = \boxed{2{,}67\ u^3}$$