TEMA 5: ANÁLISIS
Derivadas inmediatas

1. Halla la función derivada de:

a) $f(x)=3x^4-2x+5$ b) $f(x)=e^x$ c) $f(x)=2x^3-x^2+1$ d) $f(x)=\ln x$

e) $f(x)=2x^5+\dfrac{x}{3}$ f) $f(x)=\operatorname{sen} x$ g) $f(x)=x^3-3x^2+\dfrac{1}{5}$ h) $f(x)=\cos x$

i) $f(x)=4x^3-3x^2+2$ j) $f(x)=\operatorname{tg} x$ k) $f(x)=\dfrac{x^2+2}{2x+1}$ l) $f(x)=xe^x$

m) $f(x)=\dfrac{3x-1}{x^2-2}$ n) $f(x)=x^2\operatorname{sen} x$ ñ) $f(x)=\dfrac{1-x^2}{x-3}$ o) $f(x)=x\ln x$

p) $f(x)=\sqrt{x}+\dfrac{2}{x}$ q) $f(x)=\dfrac{3x+1}{e^x}$ r) $f(x)=\dfrac{3x^2}{2x+3}$ s) $f(x)=\sqrt[3]{x}\cdot\operatorname{sen} x$

2. Halla la función derivada de:

a) $f(x)=(3x^2+x)^4$ b) $f(x)=\sqrt{4x^3+1}$ c) $f(x)=e^{4x^3-2x}$ d) $f(x)=\ln(3x^4-2x)$

e) $f(x)=\operatorname{sen}\left(\dfrac{x+1}{2x-3}\right)$ f) $f(x)=3x^4-\dfrac{9x^2}{3}$ g) $f(x)=\dfrac{3x^2-2}{x^2-1}$ h) $f(x)=xe^x$

i) $f(x)=8x^5-2x^3+\dfrac{1}{3}$ j) $f(x)=(x^4-3x)e^x$ k) $f(x)=\operatorname{sen}\left(\dfrac{x}{x^2-1}\right)$ l) $f(x)=\dfrac{3x^4}{2}-\dfrac{6x^3}{5}$

m) $f(x)=\dfrac{x^2-3}{2x^3+1}$ n) $f(x)=\ln(x^4-2x)$ ñ) $f(x)=\dfrac{-2x^4+3x^2}{5}$ o) $f(x)=\dfrac{3x-4}{x^2+3x}$

p) $f(x)=\sqrt{2x^3-3}$ q) $f(x)=\dfrac{1}{2}x^4-\dfrac{3}{5}x^7$ r) $f(x)=e^x\cdot\operatorname{sen}x$ s) $f(x)=\cos\left(\dfrac{3x}{x^2+2}\right)$

t) $f(x)=4x^5-\dfrac{2x}{3}$ u) $f(x)=(x^2-3x)e^x$ v) $f(x)=\operatorname{sen}\left(\dfrac{x-1}{x^2+1}\right)$

w) $f(x)=-x^7+\dfrac{3}{4}x-1$ x) $f(x)=\dfrac{4x^3-3}{x^2-1}$ y) $f(x)=e^{7x^4-3}$

z) $f(x)=9x^2-3x^4+\dfrac{1}{3}$ 1) $f(x)=\dfrac{3x^3}{4-x^2}$ 2) $f(x)=\ln(2x^5+3x)$

3) $f(x)=\dfrac{-3x^5+2x}{7}$ 4) $f(x)=x^4\cos x$ 5) $f(x)=e^{\frac{x^2+1}{x-1}}$

6) $f(x)=\dfrac{4x^6}{3}-2x+5$ 7) $f(x)=\dfrac{2x}{x^2+1}$ 8) $f(x)=\sqrt{2x-3x^4}$

3. Halla la derivada de las siguientes funciones:

a) $f(x)=(e^x+x^5)^3$ b) $f(x)=\frac{2x}{(x+2)^2}$ c) $f(x)=(x^2+\sqrt{x})\cdot\ln x$ d) $f(x)=\ln\left(\frac{x-1}{x+2}\right)$

e) $y=e^{2x+1}\cdot\text{sen}\,x$ f) $y=\ln\left(\frac{x+3}{2x+1}\right)$ g) $y=\frac{3x+1}{(x-1)^2}$ h) $y=\cos^2(x^4-2)$

i) $f(x)=(x^2-2)\cdot e^x$ j) $f(x)=\sqrt{\frac{3x-1}{4x+2}}$

SOLUCIONES

1.

a) $f'(x)=12x^3-2$ b) $f'(x)=e^x$ c) $f'(x)=6x^2-2x$ d) $f'(x)=\frac{1}{x}$

e) $f'(x)=10x^4+\frac{1}{3}$ f) $f'(x)=\cos x$ g) $f'(x)=3x^2-6x$ h) $f'(x)=-\text{sen}x$

i) $f'(x)=12x^2-6x$ j) $f'(x)=1+tg^2x=\frac{1}{\cos^2x}$

k) $f'(x)=\frac{2x(2x+1)-(x^2+2)\cdot 2}{(2x+1)^2}=\frac{4x^2+2x-2x^2-4}{(2x+1)^2}=\frac{2x^2+2x-4}{(2x+1)^2}$ l) $f'(x)=e^x+xe^x=(1+x)e^x$

m) $f'(x)=\frac{3(x^2-2)-(3x-1)2x}{(x^2-2)^2}=\frac{3x^2-6-6x^2+2x}{(x^2-2)^2}=\frac{-3x^2+2x-6}{(x^2-2)^2}$ n) $f'(x)=2x\ \text{sen}x+x^2\cos x$

ñ) $f'(x)=\frac{-2x(x-3)-(1-x^2)}{(x-3)^2}=\frac{-2x^2+6x-1+x^2}{(x-3)^2}=\frac{-x^2+6x-1}{(x-3)^2}$ o) $f'(x)=\ln x+x\cdot\frac{1}{x}=\ln x+1$

p) $f'(x)=\frac{1}{2\sqrt{x}}-\frac{2}{x^2}$ q) $f'(x)=\frac{3e^x-(3x+1)e^x}{(e^x)^2}=\frac{e^x(3-3x-1)}{(e^x)^2}=\frac{2-3x}{e^x}$

r) $f'(x)=\frac{6x(2x+3)-3x^2\cdot 2}{(2x+3)^2}=\frac{12x^2+18x-6x^2}{(2x+3)^2}=\frac{6x^2+18x}{(2x+3)^2}$

s) $f'(x)=\frac{1}{3}x^{-2/3}\,\text{sen}\,x+x^{1/3}\cdot\cos x=\frac{1}{3\sqrt[3]{x^2}}\,\text{sen}\,x+\sqrt[3]{x}\cdot\cos x$

2.

a) $f'(x)=4\left(3x^2+x\right)^3\cdot(6x+1)$

b) $f'(x)=\dfrac{1}{2\sqrt{4x^3+1}}\cdot 12x^2=\dfrac{12x^2}{2\sqrt{4x^3+1}}=\dfrac{6x^2}{\sqrt{4x^3+1}}$

c) $f'(x)=e^{4x^3-2x}\cdot\left(12x^2-2\right)$

d) $f'(x)=\dfrac{1}{3x^4-2x}\cdot\left(12x^3-2\right)=\dfrac{12x^3-2}{3x^4-2x}$

e) $f'(x)=\cos\left(\dfrac{x+1}{2x-3}\right)\cdot\dfrac{(2x-3)-(x+1)2}{(2x-3)^2}=\dfrac{2x-3-2x-2}{(2x-3)^2}\cdot\cos\left(\dfrac{x+1}{2x-3}\right)=\dfrac{-5}{(2x-3)^2}\cdot\cos\left(\dfrac{x+1}{2x-3}\right)$

f) $f'(x)=12x^3-\dfrac{18x}{3}$

g) $f'(x)=\dfrac{6x\left(x^2-1\right)-\left(3x^2-2\right)2x}{\left(x^2-1\right)^2}=\dfrac{6x^3-6x-6x^3+4x}{\left(x^2-1\right)^2}=\dfrac{-2x}{\left(x^2-1\right)^2}$

h) $f'(x)=e^x+xe^x=e^x(1+x)$ i) $f'(x)=40x^4-6x^2$

j) $f'(x)=\left(4x^3-3\right)e^x+\left(x^4-3x\right)e^x=\left(4x^3-3+x^4-3x\right)e^x=\left(x^4+4x^3-3x-3\right)e^x$

k) $f'(x)=\cos\left(\dfrac{x}{x^2-1}\right)\cdot\dfrac{x^2-1-x\cdot 2x}{\left(x^2-1\right)^2}=\cos\left(\dfrac{x}{x^2-1}\right)\cdot\dfrac{x^2-1-2x^2}{\left(x^2-1\right)^2}=\dfrac{-x^2-1}{\left(x^2-1\right)^2}\cdot\cos\left(\dfrac{x}{x^2-1}\right)$

l) $f'(x)=\dfrac{12x^3}{2}-\dfrac{18x^2}{5}=6x^3-\dfrac{18x^2}{5}$

m) $f'(x)=\dfrac{2x\left(2x^3+1\right)-\left(x^2-3\right)6x^2}{\left(2x^3+1\right)^2}=\dfrac{4x^4+2x-6x^4+18x^2}{\left(2x^3+1\right)^2}=\dfrac{-2x^4+18x^2+2x}{\left(2x^3+1\right)^2}$

n) $f'(x)=\dfrac{1}{x^4-2x}\cdot\left(4x^3-2\right)=\dfrac{4x^3-2}{x^4-2x}$

ñ) $f'(x)=\dfrac{-8x^3+6x}{5}$

o) $f'(x)=\dfrac{3\left(x^2+3x\right)-(3x-4)(2x+3)}{\left(x^2+3x\right)^2}=\dfrac{3x^2+9x-6x^2-9x+8x+12}{\left(x^2+3x\right)^2}==\dfrac{-3x^2+8x+12}{\left(x^2+3x\right)^2}$

p) $f'(x)=\dfrac{1}{2\sqrt{2x^3-3}}\cdot 6x^2=\dfrac{6x^2}{2\sqrt{2x^3-3}}=\dfrac{3x^2}{\sqrt{2x^3-3}}$

q) $f'(x)=2x^3-\dfrac{21}{5}x^6$

r) $f'(x)=e^x\cdot \operatorname{sen}x+e^x\cdot\cos x=(\operatorname{sen}x+\cos x)e^x$

s) $f'(x) = -\text{sen}\left(\frac{3x}{x^2+2}\right)\cdot\frac{3(x^2+2)-3x\cdot 2x}{(x^2+2)^2} = -\left(\frac{3x^2+6-6x^2}{(x^2+2)^2}\right)\cdot\text{sen}\left(\frac{3x}{x^2+2}\right) =$

$= -\frac{-3x^2+6}{(x^2+2)^2}\cdot\text{sen}\left(\frac{3x}{x^2+2}\right) = \frac{3x^2-6}{(x^2+2)^2}\cdot\text{sen}\left(\frac{3x}{x^2+2}\right)$

t) $f'(x) = 20x^4 - \frac{2}{3}$

u) $f'(x) = (2x-3)e^x + (x^2-3x)e^x = e^x(2x-3+x^2-3x) = e^x(x^2-x-3)$

v) $f'(x) = \cos\left(\frac{x-1}{x^2+1}\right)\cdot\frac{x^2+1-(x-1)2x}{(x^2+1)^2} = \cos\left(\frac{x-1}{x^2+1}\right)\cdot\frac{x^2+1-2x^2+2x}{(x^2+1)^2} = \cos\left(\frac{x-1}{x^2+1}\right)\cdot\frac{-x^2+2x+1}{(x^2+1)^2} =$

$= \frac{-x^2+2x+1}{(x^2+1)^2}\cdot\cos\left(\frac{x-1}{x^2+1}\right)$

w) $f'(x) = -7x^6 + \frac{3}{4}$

x) $f'(x) = \frac{12x^2(x^2-1)-(4x^3-3)2x}{(x^2-1)^2} = \frac{12x^4-12x^2-8x^4+6x}{(x^2-1)^2} = \frac{4x^4-12x^2+6x}{(x^2-1)^2}$

y) $f'(x) = e^{7x^4-3}\cdot(28x^3) = 28x^3\cdot e^{7x^4-3}$

z) $f'(x) = 18x - 12x^3$

1) $f'(x) = \frac{9x^2(4-x^2)-3x^3(-2x)}{(4-x^2)^2} = \frac{36x^2-9x^4+6x^4}{(4-x^2)^2} = \frac{36x^2-3x^4}{(4-x^2)^2}$

2) $f'(x) = \frac{1}{2x^5+3x}\cdot(10x^4+3) = \frac{10x^4+3}{2x^5+3x}$

3) $f'(x) = \frac{-15x^4+2}{7}$

4) $f'(x) = 4x^3\cos x + x^4(-\text{sen}x) = 4x^3\cos x - x^4\text{sen}x$

5) $f'(x) = e^{\frac{x^2+1}{x-1}}\cdot\frac{2x(x-1)-(x^2+1)}{(x-1)^2} = e^{\frac{x^2+1}{x-1}}\cdot\frac{2x^2-2x-x^2-1}{(x-1)^2} = e^{\frac{x^2+1}{x-1}}\cdot\frac{x^2-2x-1}{(x-1)^2}$

6) $f'(x) = \frac{24x^5}{3} - 2 = 8x^5 - 2$

7) $f'(x) = \frac{2(x^2+1)-2x\cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$

8) $f'(x) = \frac{1}{2\sqrt{2x-3x^4}}\cdot(2-12x^3) = \frac{2-12x^3}{2\sqrt{2x-3x^4}} = \frac{2(1-6x^3)}{2\sqrt{2x-3x^4}} = \frac{1-6x^3}{\sqrt{2x-3x^4}}$

3.

a) $f'(x) = 3\,(e^x + x^5)^2 \cdot (e^x + 5x^4)$

b) $f'(x) = \dfrac{2(x+2)^2 - 2x \cdot 2(x+2)}{(x+2)^4} = \dfrac{(x+2)\,[2(x+2) - 4x]}{(x+2)^4} = \dfrac{2x+4-4x}{(x+2)^3} = \dfrac{-2x+4}{(x+2)^3}$

c) $f'(x) = \left(2x + \dfrac{1}{2\sqrt{x}}\right) \cdot \ln x + \left(x^2 + \sqrt{x}\right) \cdot \dfrac{1}{x} = \left(2x + \dfrac{1}{2\sqrt{x}}\right) \ln x + x + \dfrac{\sqrt{x}}{x}$

d) $f'(x) = \dfrac{1}{\dfrac{x-1}{x+2}} \cdot \dfrac{x+2-(x-1)}{(x+2)^2} = \dfrac{(x+2)}{(x-1)} \cdot \dfrac{(x+2-x+1)}{(x+2)^2} = \dfrac{3}{(x-1)\,(x+2)} = \dfrac{3}{x^2+x-2}$

e) $y' = e^{2x+1} \cdot 2 \cdot sen\,x + e^{2x+1} \cdot cos\,x = 2\,e^{2x+1} \cdot sen\,x + e^{2x+1} \cdot cos\,x = = e^{2x+1}\,(2sen\,x + cos\,x)$

f) $y' = \dfrac{1}{\dfrac{x+3}{2x+1}} \cdot \dfrac{2x+1-(x+3)\cdot 2}{(2x+1)^2} = \dfrac{(2x+1)}{(x+3)} \cdot \dfrac{(2x+1-2x-6)}{(2x+1)^2} = \dfrac{-5}{(x+3)\,(2x+1)} = = \dfrac{-5}{2x^2+7x+3}$

g) $y' = \dfrac{3(x-1)^2 - (3x+1)\cdot 2(x-1)}{(x-1)^4} = \dfrac{(x-1)\,[3(x-1) - 2(3x+1)]}{(x-1)^4} = \dfrac{3x-3-6x-2}{(x-1)^3} = = \dfrac{-3x-5}{(x-1)^3}$

h) $y' = 2cos\,(x^4-2) \cdot [-sen\,(x^4-2)] \cdot 4x^3 = -8x^3\,cos\,(x^4-2) \cdot sen\,(x^4-2)$

i) $f'(x) = 2x \cdot e^x + (x^2-2) \cdot e^x = (2x + x^2 - 2)\,e^x = (x^2 + 2x - 2)\,e^x$

j) $f(x) = \left(\dfrac{3x-1}{4x+2}\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(\frac{3x-1}{4x+2}\right)^{-1/2} \cdot \frac{3(4x+2)-(3x-1)\cdot 4}{(4x+2)^2} = \frac{1}{2}\left(\frac{4x+2}{3x-1}\right)^{1/2} \cdot \frac{12x+6-12x+4}{(4x+2)^2} =$$

$$= \frac{1}{2}\,\frac{(4x+2)^{1/2}}{(3x-1)^{1/2}} \cdot \frac{10}{(4x+2)^2} = \frac{5}{(3x-1)^{1/2}\cdot(4x+2)^{3/2}} = \frac{5}{\sqrt{(3x-1)\,(4x+2)^3}}$$